A NUMERICAL INVESTIGATION INTO THE LOCAL CHARACTERISTICS OF RADIATIVE HEAT EXCHANGE FOR A PAIR OF PARALLEL DISKS

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A new method is presented for the approximate solution of integral radiation equations (1), (2), and (3) in connection with the numerical determination and study of the values for the local characteristics of the radiative heat exchange between a pair of unequal parallel circular gray disks whose centers are situated on a common axis.

The results of a numerical study and from the calculation of the local boundary characteristics for radiation are presented in connection with the fundamental formulation of the problem of radiative heat exchange between a pair of parallel circular disks of different diameters, their centers situated on a common axis.

We assume the disks to be isothermal and gray; furthermore, we assume the space between these disks to be filled with a diathermic medium. The disk diameters (d_1 and d_2), the distance between the disks (h), as well as the disk temperatures (T_1 and T_2), and their emissivities (A_1 and A_2) are specified. We have to determine the density distribution for the resultant radiation over the surfaces of each of the disks.

The problem is solved by the new method of numerical (approximate) three-stage solution of integral radiation equations proposed in [1–3]:

a) the numerical determination of both the local and the mean geometric radiation angle factors between various surface elements of one disk relative to the other: $\varphi(M_iF_k)$; $M_i \in F_i$; i, k = 1, 2; $i \neq k$;

b) the numerical calculation of the local resolving radiation angle factors: $\Phi(M_i, F_k)$; $M_i \in F_i$; i, k = 1, 2; $i \neq k$;

c) the numerical determination of $E_{res}(M_i)$ over the surfaces of each of the disks.

Basic calculational formulas and expressions. The solution of the integral radiation equations in the given case reduces to the following [1–3]:

$$\theta_{res}(M_1) = -\frac{E_{res}(M_1)}{E_{0,1}} = A_1[\Phi(M_1, F_3) + A_2\Phi(M_1, F_2)\theta_{21}], \qquad (1)$$

$$\theta_{res}(M_2) = -\frac{E_{res}(M_2)}{E_{0,1}} = A_2\{\Phi(M_2, F_3) + [1 - A_2\Phi(M_2, F_2)]\theta_{21}\}, \qquad (2)$$

where

$$E_{0,k} = \sigma_0 T_k^4; \quad E_{k,i} = E_{0,k} - E_{0,i} = \sigma_0(T_k^4 - T_i^4);$$

$$\theta_{21} = \frac{T_1^4 - T_2^4}{T_1^4} = 1 - \left(\frac{T_2}{T_1}\right)^4 = 1 - \theta_2;$$

$$\theta_2 = \left(\frac{T_2}{T_1}\right)^4. \qquad (3)$$

Bearing in mind that the disks are flat, i.e., $\varphi(M_1, F_1) = \varphi(M_2, F_2) = 0$, for the local resolving radiation angle factors $\Phi(M_i, F_k)$ we derive the following calculational expressions

$$D(M_i, F_2) = D\Phi(M_i, F_2) = \varphi(M_i, F_2) + R_1\varphi_{12}\varphi(M_i, F_1) \quad (M_i \in F_i; \; i = 1, 2, 3), \qquad (4)$$

$$D(M_i, F_1) = D\Phi(M_i, F_1) = \varphi(M_i, F_1) + R_2\varphi_{21}\varphi(M_i, F_2) \quad (M_i \in F_i; \; i = 1, 2, 3), \qquad (5)$$

where

$$D = 1 - R_1R_2\varphi_{12}\varphi_{21}. \qquad (6)$$

To determine $\Phi(M_i, F_3)$, we use the expression

$$\Phi(M_i, F_3) = \varphi(M_i, F_3) + R_1\Phi(M_i, F_1)\varphi_{13} + R_2\Phi(M_i, F_2)\varphi_{23}, \qquad (7)$$

where

$$\varphi(M_1, F_3) = 1 - \varphi(M_1, F_2);$$

$$\varphi(M_2, F_3) = 1 - \varphi(M_2, F_1).$$

The results from the numerical calculations of $\Phi(M_i, F_k)$ were checked by means of closure equations of the form

$$A_1\Phi(M_i, F_1) + A_2\Phi(M_i, F_2) + \Phi(M_i, F_3) = 1 \quad (M_i \in F_i; \; i = 1, 2, 3). \qquad (8)$$

Expressions (1) and (2)—on the basis of (8)—can also be presented in the following form:

$$\theta_{res}(M_1) = A_1[1 - A_1\Phi(M_1, F_1) - A_2\Phi(M_1, F_2)\theta_2], \qquad (1a)$$

$$\theta_{res}(M_2) = A_2[1 - A_2\Phi(M_2, F_2)(2 - \theta_2) - A_1\Phi(M_2, F_1)]. \qquad (2a)$$

In the above-cited expressions F_3 is the imaginary side surface of a truncated cone with bases F_1 and F_2 for which we must assume $A_3 = 1$ and $R_3 = 0$; the temperature T_3 must be set equal to absolute zero.

Bearing in mind that $\varphi(M_1, F_1) = \varphi(M_2, F_2) = 0$, from expressions (4), (5), and (7) we obtain the following calculational expressions:

$$\Phi(M_1, F_2) = \frac{\varphi(M_1, F_2)}{1 - R_1R_2\varphi_{12}\varphi_{21}}, \qquad (9)$$

$$\Phi(M_1, F_1) = \frac{R_2\varphi_{21}\varphi(M_1, F_2)}{1 - R_1R_2\varphi_{12}\varphi_{21}} = R_2\varphi_{21}\Phi(M_1, F_2), \quad (10)$$

$$\Phi(M_2, F_1) = \frac{\varphi(M_2, F_1)}{1 - R_1R_2\varphi_{12}\varphi_{21}}, \quad (11)$$

$$\Phi(M_2, F_2) = \frac{R_1\varphi_{12}\varphi(M_2, F_1)}{1 - R_1R_2\varphi_{12}\varphi_{21}} = R_1\varphi_{12}\Phi(M_2, F_1), \quad (12)$$

$$\Phi(M_1, F_3) = 1 - \varphi(M_1, F_2) + R_1\varphi_{13}\Phi(M_1, F_1) + R_2\varphi_{23}\Phi(M_2, F_2), \quad (13)$$

$$\Phi(M_2, F_3) = 1 - \varphi(M_2, F_1) + R_1\varphi_{13}\Phi(M_2, F_1) + R_2\varphi_{23}\Phi(M_2, F_2). \quad (14)$$

The numerical calculations of $\Phi(M_i, F_k)$ can be checked by means of the expressions

$$\Phi(M_1, F_3) = 1 - A_1\Phi(M_1, F_1) - A_2\Phi(M_1, F_2), \quad (15)$$

and

$$\Phi(M_2, F_3) = 1 - A_1\Phi(M_2, F_1) - A_2\Phi(M_2, F_2). \quad (16)$$

Determination of the geometric local radiation angle factors $\varphi(M_i, F_k)$. To determine $\varphi(M_i, F_k)$ from the surface element of one disk to the other parallel disk, we use the formula

$$\varphi(M_1, F_2) = \frac{1}{2}\left[1 - \frac{h_1^2 + a_1^2 - 1}{\sqrt{(h_1^2 + a_1^2 + 1)^2 - 4a_1^2}}\right], \quad (17)$$

where $h_1 = h/r_2$ and $a_1 = a/r_2$. Here a is the instantaneous coordinate of the point M_1 which defines the position of the area dF_{M_1}, and h is the distance between the disks. The numerical calculations of $\varphi(M_i, F_k)$ (i, k = 1, 2; i ≠ k) are presented for the case in which the ratio d_2/d_1 of the disk diameters is equal to two, with the values of $D(M_1, F_2) = \varphi(M_1, F_2)$ having been found as functions of the quantity $a_1 = a/r_2$, ranging from zero to 0.5 for the two values of the parameters $h_1 = h/r_2 = 1, 2$ and as a function of $a_1 = a/r_2$, ranging from zero to two for values of $h_1 = h/r_2 = 2, 4$. These functions are shown in Fig. 1 from which it follows that with an increase in $a_1 = a/r_2$ the values of $\varphi(M_i, F_k)$ (i, k = 1, 2; i ≠ k) diminish slightly. Analogous relationships exist for $\Phi(M_2, F_1)$. The relationship between $\Phi(M_1, F_3)$ and $a_1 = a/r_2$ for various values of $h_1 = h/r_2$ and of A_1 and A_2 are shown in Fig. 1.

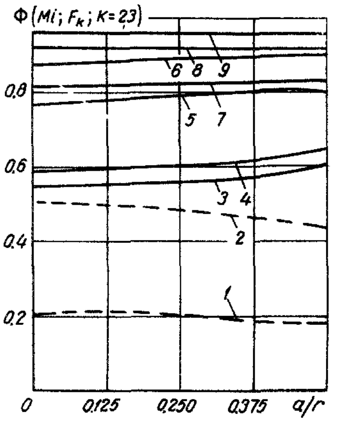

Fig. 1. Local resolving and local geometric angle coefficients of radiation versus dimensionless coodinate for various values of $h/r_2 = 2$; 1) $h/r_2 = 1$; 2) $h/r_2 = 2$; 1, 2) $\Phi(M_1, F_2)$ and $\varphi(M_1, F_2)$ coincide; 3, 4, 5, 6) $h/r_2 = 2$; 3, 7) $A_1 = A_2 = 0.9$; 4) $A_1 = A_2 = 0.8$; 5, 8; $A_1 = A_2 = 0.4$; 6, 9) $A_1 = A_2 = 0.2$.

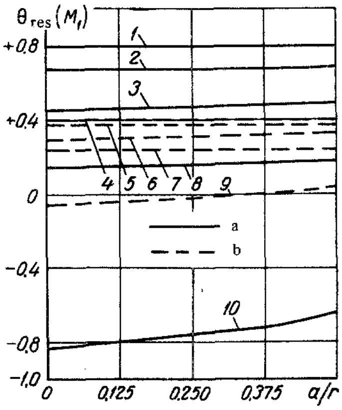

Fig. 2. Dimensionless density of resulting radiation versus dimensionless coodinate a/r for various geometric factors h/r: a) $A_1 = A_2 = 0.8$; b) $A_1 = A_2 = 0.4$; 1, 3, 4, 6, 9, 10) $h/r_2 = 1$; 1, 4) $\theta_2 = 0$; 2, 3, 5, 6) $\theta_2 = 1.5$; 1, 2, 4, 5, 7, 8) $h/r_2 = 2$.

However, of greatest interest is the dimensionless density $\theta_{res}(M_1)$ of the resultant radiation as a function of $a_1 = a/r_2$ for the various values of A_1, A_2, and θ_2, presented in Figs. 2, 3, and 4.

Here the emissivities (A_1 and A_2) of the disks were assumed to have the following values: $A_1 = A_2 = 0.1$, 0.2, 0.4, 0.6, 0.8, 0.9; b) $A_1 = 0.8$; $A_2 = 0.7$, 0.6, 0.4.

The dimensionless temperature factor θ_2 was, respectively, assumed to be equal to

$$\theta_2 = \left(\frac{T_2}{T_1}\right)^4 = 0,\ 0.1,\ 0.3,\ 0.5,\ 0.7,\ 1.0,\ 1.5,\ 2.0,\ 3.0\,^{*)}.$$

Analysis of the calculational results. Figure 2 shows the dimensionless density $\theta_{res}(M_1)$ for the resultant radiation as a function of the dimensionless coordinates a/r over the radius of disk 1 for various values of $\theta_2 = (T_2/T_1)^4$ and $A_1 = A_2$. As is clearly shown by the graphs, for various values of θ_2 and $A_1 = A_2$, $\theta_{res}(M_1)$ is minimum in value at the center of

*We note that $\theta_2 = 1$ and $\theta_2 = 0$ pertain, respectively, to the cases in which the disks exhibit identical temperature ($\theta_2 = 1$) and to the case in which the temperature of one of the disks is equal to zero ($\theta_2 = 0$).

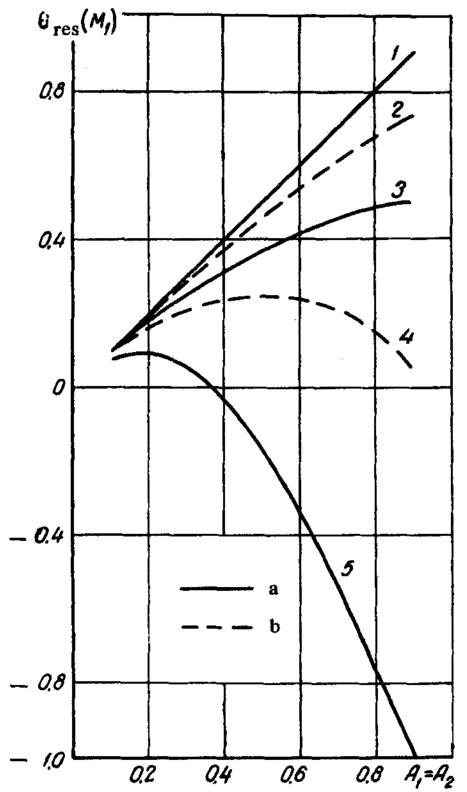

Fig. 3. Dimensionless density of resulting radiation versus absorption coefficients for various geometric factors h/r: a) $h/r_2 = 1$; b) $h/r_2 = 2$; 1) $\theta_2 = 0$ ($h/r_2 = 1$ and $h/r_2 = 2$ coincide); 2, 3) $\theta_2 = 1$; 4, 5) $\theta_2 = 1.5$.

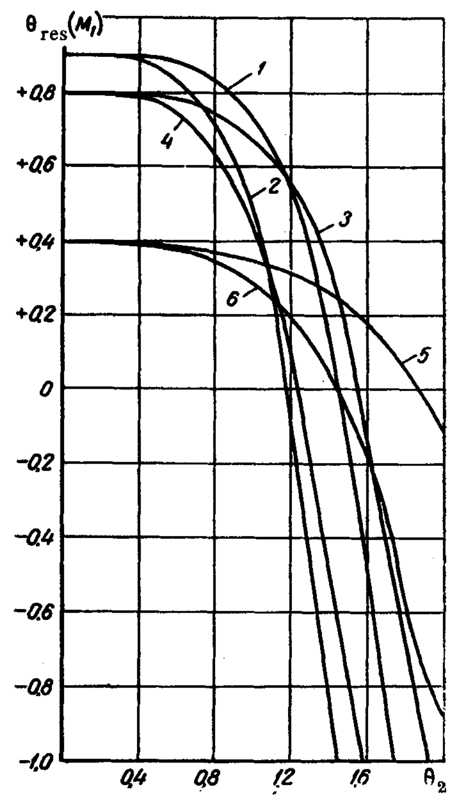

Fig. 4. Dimensionless density of resulting radiation versus dimensionless temperature parameter θ_2 for various h/r: 1, 3, 5) $h/r_2 = 2$; 2, 4, 6) $h/r_2 = 1$; 1, 2) $A_1 = A_2 = 0.9$; 3, 4) $A_1 = A_2 = 0.8$; 5, 6) $A_1 = A_2 = 0.4$.

the disk, increasing slightly with approach to the peripheral points. When $A_1 = A_2 = 0.8$; $h/r_2 = 1$ and $\theta_2 = 1.5$, the change in $\theta_{res}(M_1)$ becomes more pronounced, as we can see from curve 10 shown in Fig. 2.

The value of $\theta_{res}(M_1)$ at the various points of the disk is virtually constant for fixed values of θ_2 and $A_1 = A_2$; this is explained by the great distance $h/r_2 = 2$ between the disks. For $h/r_2 = 1$, the indicated relationship is all the more pronounced and it is the greater, the higher the value of $\theta_2 = (T_2/T_1)^4$. It should be noted here that even when $\theta_2 = 1.5$ and $h/r_2 = 1$ for $A_1 = A_2 = 0.8$ the values of $\theta_{res}(M_1)$ become negative at all points on the disk surface, whereas, at values of $\theta_2 = 0,\ 1.0$, the values of $\theta_{res}(M_1)$ are positive. For values of the emissivities A_1 and A_2 equal to 0.38, the function $\theta_{res}(M_1)$ also exhibits a positive value when $\theta_2 = 1.5$. Since $\theta_{res}(M_1)$ varies only slightly over the disk surface, it becomes of interest to find $\theta_{res}(M_1)$ at certain fixed points M_1 of the disk as a function of θ_2 for various values of A_1 and A_2, as well as a function of A_1 and A_2 for various values of θ_2, as shown in Figs. 3 and 4.

We see from these figures than when $d_2/d_1 = 2$ and $h/r_2 = 1$, the dimensionless density $\theta_{res}(M_1)$ of the resultant radiation at the point M_1 ($a/r = 1/4$) with an increase in $A_1 = A_2$ increases all the more, the smaller the value of θ_2. However, even when $\theta_2 = 1.5$, we have a diminishing function which from $A_1 = A_2 = 0.4$ on can even assume negative values. Figure 3 also shows curves demonstrating this relationship for $h/r_2 = 2$.

It is essential that we point out (see Fig. 4) that if the value of $\theta_{res}(M_1)$ remains virtually constant for a change in θ_2 from zero to 0.5, with a further increase in θ_2 the resulting reduction in $\theta_{res}(M_1)$ is the greater, the larger θ_2 and $A_1 = A_2$, so that for certain specific values of θ_2 (which are the smaller, the larger $A_1 = A_2$) the function $\theta_{res}(M_1)$ become negative.

In conclusion, we note that for the special case of two equal disks with identical emissivities, the numerical calculations of the local radiation characteristics were first carried out by means of Sparrow computers [7]. However, the computational method employed in [7] is marked by the fact that it is cumbersome and excessively complex, which was pointed out, in particular, in reference [5].

A systematic investigation has been undertaken in this paper and a solution derived for the problem associated with the determination of local radiation characteristics for the more general case of unequal disks with various temperature and emissivity values.

NOTATION

$\varphi(M_i, F_k)$ and $\Phi(M_i, F_k)$ are the local geometric and resolving angle factor for the radiation from point M_i of surface F_i to surface F_k; φ_{ik} is the mean geometric radiation angle factor from surface F_i to surface F_k; $E_{res}(M_i)$ and $\theta_{res}(M_1)$ are the local and dimensionless densities of the resulting radiation at point M_i of surface F_i.

REFERENCES

1. Yu. A. Surinov, Izv. AN SSSR, Energetika i transport, no. 5, 1965.
2. Yu. A. Surinov, Chernaya metallurgiya, no. 3, 1966.
3. Yu. A. Surinov, Izv. AN SSSR, Energetika i transport, no. 2, 1967.
4. Yu. A. Surinov, "Heat transfer and thermal simulation," collection: Transactions of the Krzhizhanovskii Institute of Power Engineering [in Russian], Izd. AN SSSR, 1959.
5. Yu. A. Surinov and O. P. Luk'yanenko, Izv. AN SSSR, Energetika i transport, no. 3, 1967.
6. E. R. Eckert and R. M. Drake, The Theory of Heat and Mass Transfer [Russian translation], Gosenergoizdat, 1961.
7. Sparrow and Gregg, Transactions of the ASME, Ser. C: Journal of Heat Transfer, Vol. 83, no. 4, [Russian translation], IL, 1961.

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